# Maths Refresher

## Workbook 1

This booklet will help to refresh your understanding of:

- number
- decimals, fractions and percentages
- exponents and roots

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# Maths Refresher

# Booklet 1

## Contents

# 1. Introduction: Number

- Mathematics is the science of patterns and relationships related to quantity. Listed below are some of the many relationships you may have come across:

  - Every number is related to every other number in a number relationship. For example, 8 is 2 less than 10; made up of 4 and 4 (or 3 and 5); and is 10 times 0.8; is the square root of 64; and so on….
  - Number relationships are the foundation of strategies that help us remember number facts. For instance, knowing 4 + 4 = 8 allows one to quickly work out 4 + 5 = 9 (one more than 8); If one knows that 2 x 5 = 10, then 4 x 5 and 8 x 5 can easily be calculated (double 2 is 4 and so double 10 is 20; then double 4 is 8 and so double 20 is 40).
  - Each digit in a written numeral has a 'place' value which shows its relationship to '1'. For example, in 23.05 the value of the '2' is 20 ones, while the value of the '5' is only five-hundredths of one. Understanding place value is critical to working with numbers.

- Mathematics is considered a universal language; however, words in English can often have more than one meaning which is why we sometimes find it difficult to translate from English to mathematical expressions.

- Arithmetic is a study of numbers and their manipulation.

- The most commonly used numbers in arithmetic are integers, which are positive and negative whole numbers including zero. For example: -6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6. Decimal fractions are not integers because they are 'parts of a whole', for instance, 0.6 is 6 tenths of a whole.

- The symbols we use between the numbers to indicate a task or relationships are the **operators**, and the table below provides a list of common operators. You may recall the phrase, 'doing an operation.'

| Symbol | Meaning |
|--------|---------|
| $+$ | Add, Plus, Addition, Sum |
| $-$ | Minus, Take away, Subtract, Difference |
| $\times$ | Times, Multiply, Product, |
| $\div$ | Divide, Quotient |
| $\pm$ | Plus or Minus |
| $\|a\|$ | Absolute Value (ignore –ve sign) |
| $=$ | Equal |
| $\neq$ | Not Equal |
| $<$ | Less than |
| $>$ | Greater than |
| $\ll$ | Much Less than |
| $\gg$ | Much More than |
| $\approx$ | Approximately equal to |
| $\leq$ | Less than or equal |
| $\geq$ | Greater than or equal |

**Example:**
1. $12 \pm 6 = 6 \; or \; 18. \quad 12 - 6 \; or \; 12 + 6$
2. $|-6| = 6$
3. $12.999 \approx 13$
4. $12 \neq 7$
5. Speed of light $\gg$ speed of sound

**1. Your Turn:**
Are the following statements true?
a) $32 \neq 4 \times 8$
b) $7 > 6$
c) $4 \geq 4$
d) $|5| = 5$
e) $37.1 + 22.02 = 59.3$

## Integers

Whole numbers are integers; there are positive and negative integers. Positive integers are 1, 2, 3, 4, 5…
The negative integers are … -5, -4, -3, -2, -1 (the dots before or after the sequence indicate that there are more numbers in this sequence that continue indefinitely).

Here are some more terms for you:

An **equation** implies that what is on either side of the **'='** sign balances.
The **sum** of two numbers implies two numbers are added together.



> The sum of 4 and 8 is 12; $4 + 8 = 12$

The **difference** of two numbers implies that the second number is subtracted from the first number.

> The difference between 9 and three is 6; $9 - 3 = 6$

The **product** of two numbers implies that two numbers are multiplied together.

> The product of 3 and 4 is 12; $3 \times 4 = 12$

The **quotient** of two numbers implies that the first number is divided by the second.

> The quotient of 20 and 4 is 5; $\frac{20}{4} = 5$ or $20 \div 4 = 5$

**Rational Number:** The term rational derives from the word ratio. Hence, a rational number can be a described by a ratio of integers or as a fraction. For example, $\frac{3}{4}$ $and$ $0.75$ are both rational numbers.
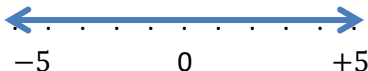
**Irrational number**: A number that cannot be written as a simple fraction or as a decimal fraction. If the number goes on forever without **terminating**, and without **repeating**, then it is an irrational number. For example, $\pi$ is a recurring decimal that does not repeat: 3.14159… Therefore, $\pi$ is an irrational number.

## Directed Numbers (negative and positive integers)

Directed numbers are numbers that have positive $(+)$ and $(-)$ signs signifying their direction.
Note that when using the calculator, we use the $(-)$ key rather than the subtraction key, and each negative number may need to be bracketted, for instance, $(-3) + (+3) = 0$.
When naming directed numbers we use the terms *negative and positive* numbers; avoiding the terms *plus* and *minus* unless you are indicating that an operation is taking place of plus for addition and minus for subtraction. So, $(-3) + 3 = 0$ reads *'negative 3 plus positive 3 equals zero'*
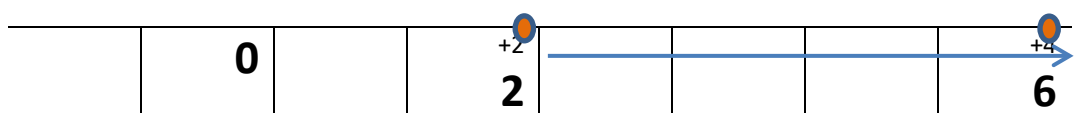To use a graphic symbol we can display $(-5)$ and $(+5)$ as

$$-5 \qquad 0 \qquad +5$$

This graphic symbol is known as a number line and can be used to show how and why operations work.
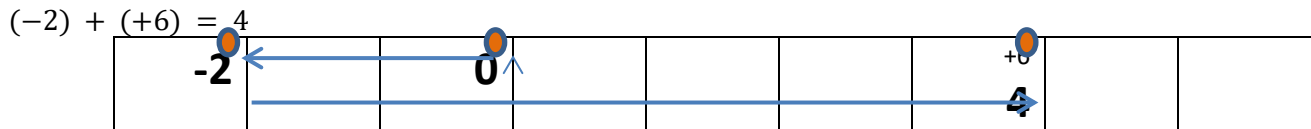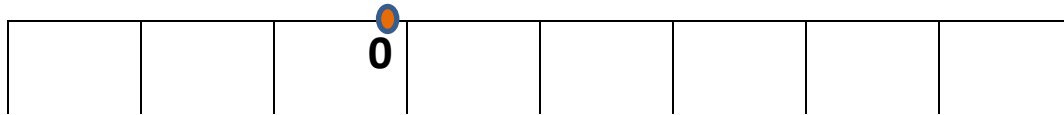
## Addition:
To add a number we move to the right:
$2 + 4 = 6$

Here we are adding a positive number beginning at a negative number. Thus if we begin at zero we move in a negative direction to get to -2, and then in a positive direction of 6, and so we reach +4:

$$(-2) + (+6) = 4$$



Your Turn: Represent $-3 + 5 =$



**Subtraction:**
To subtract a positive number we move that number of places to the left. For example, $5 - 7$ means we are subtracting a positive number, so we start at five and move 7 places to the left to get $-2$. $\therefore 5 - 7 = -2$

Then to subtract a negative number we do the opposite and so we move to the right.

For example: $(-2) - (-5) = 3$ 'negative 2 minus negative 5' which means $(-2) + 5$. Hence, subtracting a negative number is the same as adding a positive)



Remember that subtraction of an integer means adding its opposite, thus, if we subtract a negative number we move to the right and if we add a negative number we move to the left – the opposite. For example, if we add a negative number 5+(-3) then we move to the left:
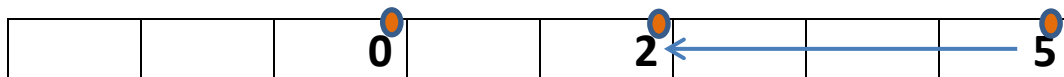


So, $5 + (-3)$ is the same as $5 - 3 = 2$ (so adding a negative number is the same as subtracting a positive number)

**1. Your Turn:**

    f)   What is the value of 6 in the number 896 324.51

    g)   What is the value of 1?

    h)   What is the number that is six more than the difference between nineteen and ten.

    i)   From the product of twelve and six, subtract the quotient of twelve and six

    j)   Evaluate (i)    $8 + (-4) =$

                  (ii) $-15 + (-6) =$

                  (iii) $-15 - (-6) =$

# 2. Rounding and Estimating

Rounding numbers is a method of decreasing the accuracy of a number to make calculations easier. Rounding is important when answers need to be given to a particular degree of accuracy. With the advent of calculators, we also need to be able to estimate a calculation to detect when the answer might be incorrect.

**The Rules for Rounding:**

1. Choose the last digit to keep.
2. If the digit to the right of the chosen digit is 5 or greater, increase the chosen digit by 1.
3. If the digit to the right of the chosen digit is less than 5, the chosen digit stays the same.
4. All digits to the right are now removed.

   For example, what is 7 divided by 9 rounded to 3 decimal places?
   So, $7 \div 9 = 0.777777777777777777777777777777777777etc.$
   The chosen digit is the third seven (3 decimal places).
   The digit to the right of the chosen digit is 7, which is larger than 5, so we increase the 7 by 1, thereby changing this digit to an 8.
   $\therefore 7 \div 9 = 0.778$ to three decimal places.

5. The quotient in rule 4 above is called a **recurring decimal**. This can also be represented as $0.\dot{7}$; the dot above signifies that the digit is repeated. If the number was 0.161616, it would have two dots to symbolise the two repeating digits: $0.\dot{1}\dot{6}$

**Estimating** is a very important ability which is often ignored. A leading cause of getting math problems wrong is because of entering the numbers into the calculator incorrectly. It helps to be able to estimate the answer to check if your calculations are correct.

Some simple methods of estimation:

- **Rounding:** $273.34 + 314.37 = ?$ If we round to the tens we get $270 + 310$ which is much easier and quicker. We now know that $273.34 + 314.37$ should equal approximately 580.
- **Compatible Numbers:** $527 \times 12 = ?$ If we increase 527 to 530 and decrease 12 to 10, we have $530 \times 10 = 5300$. A much easier calculation.
- **Cluster Estimation:** $357 + 342 + 370 + 327 = ?$ All four numbers are clustered around 350, some larger, some smaller. So we can estimate using $350 \times 4 = 1400$.

**Example Problems:**

1. Round the following to 2 decimal places:
   a. 22.6783   gives   22.68
   b. 34.6332   gives   34.63
   c. 29.9999   gives   30.00
2. Estimate the following:
   a. $22.5684 + 57.355 \approx 23 + 57 = 80$
   b. $357 \div 19 \approx 360 \div 20 = 18$
   c. $27 + 36 + 22 + 31 \approx = 30 \times 4 = 120$

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**"Rounding decimals: to the nearest tenth"**
https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/dec-rounding-estimation-pre-alg/v/rounding-decimals
**"Multiplication estimation example"**
https://www.khanacademy.org/math/arithmetic/multiplication-division/multi_digit_multiplication/v/multiplying-whole-numbers-and-applications-4

**2. Your Turn:**

A. Round the following to 3 decimal places:
   a. $34.5994 \approx$
   b. $56.6734 \approx$

B. Estimate the following:
   a. $34 \times 62 \approx$
   b. $35.9987 - 12.76 \approx$
   c. $35 + 32 + 27 + 25 \approx$

# 3. Order of Operations

The order of operations matters when solving equations. Look at the example: $3 + 6 \times 2 = ?$

If I do the addition, then the multiplication, the answer would be: $9 \times 2 = 18$
If I do the multiplication, then the addition, the answer would be: $3 + 12 = 15$
There cannot be two answers to the same question. A rule is required to make sure everyone uses the same order.

There is a **Calculation Priority Sequence** to follow. Different countries, different states, even different teachers use a different mnemonic to help you remember the order of operations, but two common versions are BOMDAS and BIMDAS which stand for:

| | | |
|---|---|---|
| **B**rackets | | $\{[( \quad )]\}$ |
| **O**ther or **I**ndices | | $x^2, \sin x, \ln x, etc$ |
| **M**ultiplication or **D**ivision | | $\times \ or \ \div$ |
| **A**ddition or **S**ubtraction | | $+$ or $-$ |

## The Rules:

1. Follow the order (BIMDAS, BOMDAS or BODMAS)
2. If two operations are of the same level, you work from left to right. E.g. $(\times \ or \ \div) \ or \ (+ \ or \ -)$
3. If there are multiple brackets, work from the inside set of brackets outwards. $\{[( \ )]\}$

## Example Problems:

1. Solve: $\qquad\qquad\qquad\qquad 5 + 7 \times 2 + 5^2 =$

   Step 1: $5^2$ has the highest priority so: $\quad 5 + 7 \times 2 + 25 =$

   Step 2: $7 \times 2$ has the next priority so: $\quad 5 + 14 + 25 =$

   Step 3: only addition left, thus left to right: $19 + 25 = 44$

   $$\therefore 5 + 7 \times 2 + 5^2 = 44$$

2. Solve: $\qquad\qquad [(3 + 7) \times \ 6 - 3] \times 7 =$
   $$[10 \times \ 6 - 3] \times 7 =$$
   $$[ \ 60 - 3] \times 7 =$$
   $$57 \ \times 7 = 399$$
   $$\therefore [(3 + 7) \times \ 6 - 3] \times 7 = 399$$

## 3. Your Turn:

a) $4 \times (5 + 2) + 6 - 12 \div 4 =$

b) $3 \times 7 + 6 - 2 + 4 \div 2 + 7 =$

c) $2.4 - 0.8 \times 5 + 8 \div 2 \times 6 =$

d) $(2.4 - 0.8) \times 5 + 8 \times 6 \div 2 =$

# 4. Naming Fractions

- Fractions are representations of "even parts of a whole."
- A key concept is that division and fractions are linked. Even the division symbol ( ÷ ) is a fraction.

$$\frac{1}{2} \; is \; the \; same \; as \; 1 \; divided \; by \; 2 \; which \; is \; 0.5.$$

- A fraction is made up of two main parts: $\frac{3}{4}$  →  $\frac{Numerator}{Denominator}$

The **denominator** represents how many parts of the whole there are, and the **numerator** indicates how many of the parts are of interest.

For instance, $\frac{5}{8}$ of a pie means that we have cut a pie into 8 **even** pieces and we are interested in the five that are left on the plate.
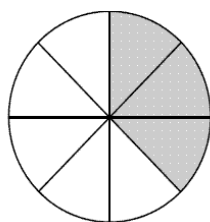


- Fractions should always be displayed in their simplest form. For example, $\frac{6}{12}$ is written as $\frac{1}{2}$

Strategies for converting fractions into their simplest form will be covered over the next sections.

- A **proper fraction** has a numerator smaller than the denominator, for example, $\frac{3}{4}$
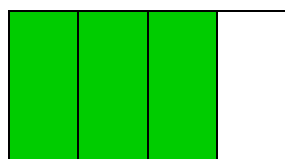
 This representation shows that we have four equal parts and have shaded three of them, therefore $\frac{3}{4}$

- An **improper fraction** has a numerator larger than the denominator, for example, $\frac{4}{3}$

 Here we have two 'wholes' divided into three equal parts.

Three parts of '3 equal parts' makes a 'whole' plus one more part makes '*one whole and one third*' or '*four thirds*'

- Therefore, a **mixed fraction** has a whole number and a fraction, for example, $1\frac{1}{3}$

## 4. Your Turn:

Name the fractions:

a) What fraction of the large square is black?

b) What fraction of the large square has vertical lines?

c) What fraction of the large square has diagonal lines?

d) What fraction of the large square has wavy lines?
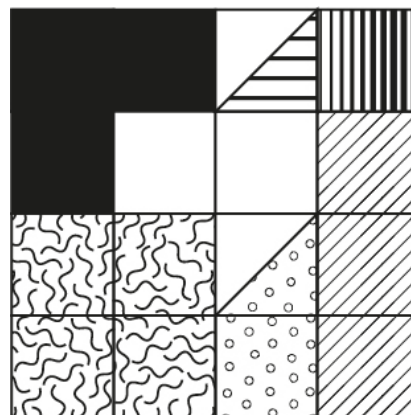
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# 5. Equivalent Fractions

Equivalence is a concept that is easy to understand when a fraction wall is used.



As you can see, each row has been split into different fractions: top row into 2 halves, bottom row 12 twelfths. An equivalent fraction splits the row at the same place. Therefore:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$$

The more pieces I split the row into (denominator), the more pieces I will need (numerator).

Mathematically, whatever I do to the numerator (multiply or divide), I must also do to the denominator and vice versa, whatever I do to the denominator I must do to the numerator. Take $\frac{2}{3}$ as an example. If I multiply the numerator by 4, then I must multiply the denominator by 4 to create an equivalent fraction:

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

**Example problems:** Use what you have understood about equivalent fractions to find the missing values in these fraction pairs.

1. $\frac{3}{5} = \frac{}{20}$  Answer: The denominator was multiplied by 4. (20 ÷5 =4)

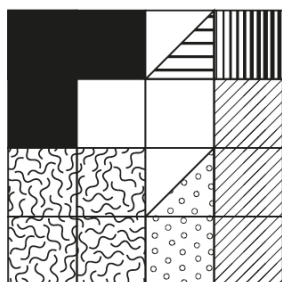   So the numerator must by multiplied by 4.   $\therefore \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$

2. $\frac{27}{81} = \frac{9}{}$  Answer: The numerator was divided by 3. (27 ÷9 =3)

   So the denominator must be divided by 3.   $\therefore \frac{27 \div 3}{81 \div 3} = \frac{9}{27}$

## 5. Your Turn:

a)  $\frac{2}{3} = \frac{}{9}$

b)  $\frac{5}{7} = \frac{45}{}$

c)  $\frac{9}{10} = \frac{}{30}$

d)  $\frac{}{52} = \frac{4}{13}$



Return to Your Turn Activity 4.

e) What fraction of the large square has dots?

f) What fraction of the large square has horizontal lines?

# 6. Converting Mixed Numbers to Improper Fractions

A mixed number is a way of expressing quantities greater than 1. A mixed number represents the number of wholes and remaining parts of a whole that you have, while an improper fraction represents how many parts you have. The diagram below illustrates the difference between a mixed number and an improper fraction, using a quantity of car oil as an example. On the left, we use a mixed number to represent 3 whole litres and 1 half litre. We write this mixed number as 3 ½. On the right, we use an improper fraction to represent 7 half litres. We write this improper fraction as $\frac{7}{2}$.



Is the same as
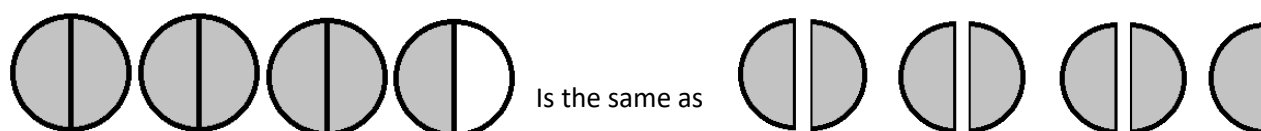
$$3\frac{1}{2} = 7\ halves = \frac{7}{2}$$

You are more likely to encounter mixed numbers than improper fractions in everyday language. For example, you are more likely to say, 'my car requires 3 ½ litres of oil,' rather than, 'my car requires $\frac{7}{2}$ litres of oil.'

It is much easier to multiply or divide fractions when they are in improper form. As such, mixed numbers are usually converted to improper fractions before they are used in calculations. To convert from a mixed number to an improper fraction, multiply the whole number by the denominator then add the numerator. This total then becomes the new numerator which is placed over the original denominator. For example:

Convert $3\frac{1}{2}$ into an improper fraction.

working:   $3(whole\ number) \times 2(denominator) + 1(numerator) = 7$

Therefore, $the\ improper\ fraction\ is\ \frac{7}{2}$

**Example problems:**

1.  $2\frac{2}{3} = \frac{8}{3}\ Note: (2 \times 3 + 2 = 8)$

2.  $2\frac{3}{7} = \frac{17}{7}\ Note: (2 \times 7 + 3 = 17)$

**6. Your Turn:**

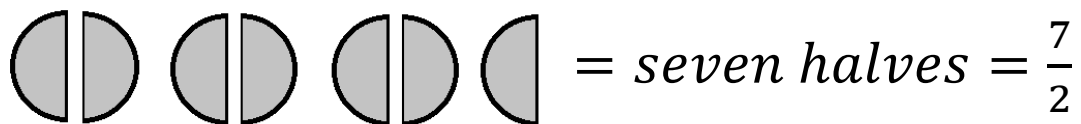Convert these mixed numbers to improper fractions.

a)   $4\frac{1}{2} = —$

b)   $5\frac{1}{3} = —$

c)   $7\frac{3}{5} = —$

d)   $2\frac{1}{8} = —$

# 7. Converting Improper Fractions to Mixed Numbers

While improper fractions are good for calculations, they are rarely used in everyday situations. For example, people do not wear a size $\frac{23}{2}$ shoe; instead they wear a size $11\frac{1}{2}$ shoe.



$$= seven\ halves = \frac{7}{2}$$

To convert to an improper fraction we need to work out how many whole numbers we have. Here we reverse the procedure from the previous section. We can see that 6 of the halves combine to form 3 wholes; with a half left over.



$$= 3\frac{1}{2}$$
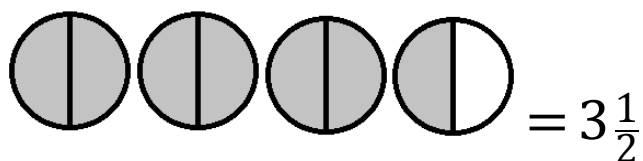
So to work this symbolically as a mathematical calculation we simply divide the numerator by the denominator. Whatever the remainder is becomes the new numerator.

> Using a worked example of the diagram above: Convert $\frac{7}{2}$

$7 \div 2 = 3\frac{1}{2}$ If I have three whole numbers, then I also have six halves and we have one half remaining. $\therefore \frac{7}{2} = 3\frac{1}{2}$

**That was an easy one. Another example:**

Convert $\dfrac{17}{5}$ into a *mixed fraction.*

> working: $17 \div 5 =$ the whole number is 3 with some remaining.

> If I have 3 whole numbers that is 15 $fifths.\,(3 \times 5)$ I must now have 2 $fifths$ remaining. $(17 - 15)$

> Therefore, I have $3\frac{2}{5}$

**Example problems:** Convert the improper fractions to mixed numbers:

1. $\frac{27}{6} = 4\frac{3}{6} = 4\frac{1}{2}$
   $Note: (27 \div 6 = 4.5)\ so\ (4 \times 6 = 24)\ with\ (27 - 24 = 3)$ remaining. Don't forget equivalent fractions.

2. $\frac{8}{3} = 2\frac{2}{3}$ $Note: (8 \div 3 = 2.67)\ so\ (2 \times 3 = 6)\ with\ (8 - 6 = 2)$ remaining.

## 7. Your Turn:

Convert the following improper fractions to mixed numbers:

a) $\frac{7}{5} =$                          c) $\frac{53}{9} =$

b) $\frac{12}{9} =$                         d) $\frac{27}{7} =$

# 8. Converting Decimals into fractions

Decimals are an almost universal method of displaying data, particularly given that it is easier to enter decimals, rather than fractions, into computers. But fractions can be more accurate. For example, $\frac{1}{3}$ is not $0.33$ it is $0.3\dot{3}$

The method used to convert decimals into fractions is based on the notion of place value. The place value of the last digit in the decimal determines the denominator: tenths, hundredths, thousandths, and so on…

**Example problems:**

1. 0.5 has 5 in the tenths column. Therefore, 0.5 is $\frac{5}{10} = \frac{1}{2}$ (simplified to an equivalent fraction).
2. 0.375 has the 5 in the thousandth column. Therefore, 0.375 is $\frac{375}{1000} = \frac{3}{8}$
3. 1.25 has 5 in the hundredths column and you have $1\frac{25}{100} = 1\frac{1}{4}$

The hardest part is converting to the lowest equivalent fraction. If you have a scientific calculator, you can use the fraction button. This button looks different on different calculators so read your manual if unsure.

If we take $\frac{375}{1000}$ from example 2 above:

Enter 375 then  followed by 1000 press = and answer shows as $\frac{3}{8}$.

NOTE: The calculator does not work for rounded decimals; especially thirds. For example, $0.333 \approx \frac{1}{3}$

The table below lists some commonly encountered fractions expressed in their decimal form:

| Decimal | Fraction |
|---------|----------|
| 0.125 | $\frac{1}{8}$ |
| 0.25 | $\frac{1}{4}$ |
| 0.33333 | $\frac{1}{3}$ |
| 0.375 | $\frac{3}{8}$ |
| 0.5 | $\frac{1}{2}$ |
| 0.66667 | $\frac{2}{3}$ |
| .75 | $\frac{3}{4}$ |

## 8. Your Turn:

(No Calculator first, then check.)

a) 0.65 =

b) 2.666 =

c) 0.54 =

d)    3.14 =

# 9. Converting Fractions into Decimals

Converting fractions into decimals is based on place value. For example, applying what you have understood about equivalent fractions, we can easily convert $\frac{2}{5}$ into a decimal. First we need to convert to a denominator that has a 10 base. Let's convert $\frac{2}{5}$ into tenths $\rightarrow \frac{2\times2}{5\times2} = \frac{4}{10}$ $\therefore$ we can say that two fifths is the same as four tenths: 0.4

Converting a fraction to decimal form is a simple procedure because we simply use the divide key on the calculator. Note: If you have a mixed number, convert it to an improper fraction before dividing it on your calculator.

**Example problems:**

1.  $\frac{2}{3} = 2 \div 3 = 0.66666666666 \ldots \approx 0.67$
2.  $\frac{3}{8} = 3 \div 8 = 0.375$
3.  $\frac{17}{3} = 17 \div 3 = 5.6666666 \ldots \approx 5.67$
4.  $3\frac{5}{9} = (27 + 5) \div 9 = 3.555555556 \ldots \approx 3.56$

**9. Your Turn:** (Round your answer to three decimal places where appropriate)

a)  $\frac{17}{23} =$

b)  $\frac{5}{72} =$

c)  $56\frac{2}{3} =$

d)  $\frac{29}{5} =$

# 10. Fraction Addition and Subtraction

Adding and subtracting fractions draws on the concept of equivalent fractions. The golden rule is that you can only **add and subtract fractions** if they have the **same denominator**, for example, $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.

However, if two fractions do **not** have the same denominator, we must use equivalent fractions to find a "common denominator" before they can be added together.

For instance, we cannot simply add $\frac{1}{4} + \frac{1}{2}$ because these fractions have different denominators (4 and 2). As such, arriving at an answer of $\frac{2}{6}$ (two sixths) would be incorrect. Before these fractions can be added together, they must both have the same denominator.

From the image at right, we can see that we have three quarters of a whole cake. So to work this abstractly, we need to decide on a common denominator, 4, which is the lowest common denominator. Now use the equivalent fractions concept to change $\frac{1}{2}$ *into* $\frac{2}{4}$ by multiplying both the numerator and denominator by two: $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$

Now that the denominators are the same, the addition can be carried out:

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

**Let's try another:**

$\frac{1}{3} + \frac{1}{2}$ We cannot simply add these fractions; again we need to find the lowest common denominator. The easiest way to do this is to multiply the denominators: $\frac{1}{3}$ *and* $\frac{1}{2}$ (2 x 3 = 6). Therefore, both fractions can have a denominator of 6, yet we need to change the numerator. The next step is to convert both fractions into sixths as an equivalent form. How many sixths is one third? $\frac{1}{3} = \frac{2}{6}$ $\left(\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}\right)$ And how many sixths is one half? $\frac{1}{2} = \frac{3}{6}$ $\left(\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}\right)$

Therefore: $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

With practise, a pattern forms, as is illustrated in the next example:

$$\frac{1}{3} + \frac{2}{5} = \frac{(1 \times 5) + (2 \times 3)}{(3 \times 5)} = \frac{5 + 6}{15} = \frac{11}{15}$$

In the example above, the lowest common denominator is found by multiplying 3 and 5, and then the numerators are multiplied by 5 and 3 respectively.

Use the following problems to reinforce the pattern.

**10. Your Turn:**

a) $\frac{1}{3} + \frac{2}{5} =$

c) $2\frac{2}{3} + 1\frac{3}{4} =$

b) $\frac{3}{4} + \frac{2}{7} =$

d) $2\frac{1}{6} + 3\frac{7}{8} =$

**Subtraction is the same procedure but with a negative symbol:**

$$\frac{2}{3} - \frac{1}{4} = \frac{(2 \times 4) - (1 \times 3)}{(3 \times 4)}$$

$$= \frac{8 - 3}{12}$$

$$= \frac{5}{12}$$

**10. (continued) Your Turn:**

e) $\frac{9}{12} - \frac{1}{3} =$

f) $\frac{1}{3} - \frac{1}{2} =$

# 11. Fraction Multiplication and Division

Compared to addition and subtraction, multiplication and division of fractions is easy to do, but sometimes a challenge to understand how and why the procedure works mathematically. For example, imagine I have $\frac{1}{2}$ of a pie and I want to share it between 2 people. Each person gets a quarter of the pie.

$$\frac{1}{2} \, of \, \frac{1}{2} = \frac{1}{4}$$
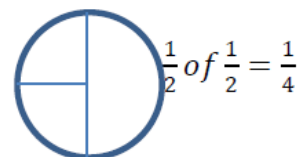
Mathematically, this example would be written as: $\quad \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Remember that fractions and division are related; in this way, multiplying by a half is the same as dividing by two.

So $\frac{1}{2}$ (two people to share) of $\frac{1}{2}$ (the amount of pie) is $\frac{1}{4}$ (the amount each person will get).

But what if the question was more challenging: $\frac{2}{3} \times \frac{7}{16} =$? This problem is not as easy as splitting pies.

A mathematical strategy to use is: "Multiply the numerators then multiply the denominators"

Therefore, $\frac{2}{3} \times \frac{7}{16} = \frac{(2\times7)}{(3\times16)} = \frac{14}{48} = \frac{7}{24}$

However, we can also apply a cancel out method – which you may recall from school. The rule you may recall is, 'What we do to one side, we must do to the other.'

Thus, in the above example, we could simplify first: $\frac{2}{3} \times \frac{7}{16} = ?$ The first thing we do is look to see if there are any common multiples. Here we can see that 2 is a multiple of 16, which means that we can divide top and bottom by 2: $\quad \frac{2^{\div2}}{3} \times \frac{7}{16_{\div2}} = \frac{1}{3} \times \frac{7}{8} = \frac{1\times7}{3\times8} = \frac{7}{24}$

**Example Multiplication Problems:**

1. $\frac{4}{9} \times \frac{3}{4} = \frac{(4\times3)}{(9\times4)} = \frac{12}{36} = \frac{1}{3}$

   Have a go at simplifying first and then perform the multiplication.


   $\frac{4^{\div4}}{9_{\div3}} \times \frac{3^{\div3}}{4_{\div4}} = \frac{(1\times1)}{(3\times1)} = \frac{1}{3}$

2. $2\frac{4}{9} \times 3\frac{3}{5} = \frac{22}{9} \times \frac{18}{5} = \frac{(18\times22)}{(9\times5)} = \frac{396}{45} = 396 \div 45 = 8.8$


   $\frac{22}{9_{\div9}} \times \frac{18^{\div9}}{5} = \frac{22\times2}{1\times5} = \frac{44}{5}$ so $44 \div 5 = 8\frac{4}{5}$

**Division of fractions seems odd, but it is a simple concept:**

You may recall the expression 'invert and multiply' which means we flip the fraction; we switch the numerator and the denominator. Hence, $\div \frac{1}{2}$ *is the same as* $\times \frac{2}{1}$

If the sign is swapped to its opposite, the fraction is flipped upside down, this 'flipped' fraction is referred to as the **reciprocal of the original fraction**.

Therefore, $\frac{2}{3} \div \frac{1}{2}$ is the same as $\frac{2}{3} \times \frac{2}{1} = \frac{(2 \times 2)}{(3 \times 1)} = \frac{4}{3} = 1\frac{1}{3}$   Note: dividing by half doubled the answer.

**Example Division Problems:**

1.  $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{(2 \times 5)}{(3 \times 3)} = \frac{10}{9} = 1\frac{1}{9}$

2.  $3\frac{3}{4} \div 2\frac{2}{3} = \frac{15}{4} \div \frac{8}{3} = \frac{15}{4} \times \frac{3}{8} = \frac{(15 \times 3)}{(4 \times 8)} = \frac{45}{32} = 1\frac{13}{32}$

**11. Your Turn:**

a)  Find the reciprocal of $2\frac{2}{5}$

b)  $\frac{2}{3} \times \frac{7}{13} =$

c)  $1\frac{1}{6} \times \frac{2}{9} =$

d)  $8 \times \frac{3}{4} \times \frac{5}{6} \times 1\frac{1}{2} =$

e)  $\frac{3}{7} \div \frac{2}{5} =$

f)  $2\frac{2}{5} \div 3\frac{8}{9} =$

g)  $\frac{(-25) \div (-5)}{4 - 2 \times 7} =$

h)  $\frac{-7}{2} \div \frac{-4}{9} =$

i)  If we multiply 8 and the reciprocal of 2, what do we get?

j)  What is 40 mulitplied by 0.2? (use your knowledge of fractions to solve)

# 12. Percentage

The concept of percentage is an extension of the material we have already covered about fractions. To allow comparisons between fractions we need to use the same denominator. As such, all percentages use 100 as the denominator. The word percent or "per cent" means per 100. Therefore, 27% is $\frac{27}{100}$.

To use percentage in a calculation, the simple mathematical procedure is modelled below:

For example, 25% of 40 is $\frac{25}{100} \times 40 = 10$

Percentages are most commonly used to compare parts of an original. For instance, the phrase '30% off sale,' indicates that whatever the original price, the new price is 30% less. However, percentages are not often as simple as, for example, 23% of 60. Percentages are commonly used as part of a more complex question. Often the questions might be, "How much is left?" or "How much was the original?"

## Example problems:

A.  An advertisement at the chicken shop states that on Tuesday everything is 22% off. If chicken breasts are normally $9.99 per kilo. What is the new per kilo price?

Step 1: SIMPLE PERCENTAGE:

$\frac{22}{100} \times 9.99 = 2.20$

Step 2: DIFFERENCE: Since the price is cheaper by 22%, $2.20 is subtracted from the original: $9.99 - 2.20 = \$7.79$

B.  For the new financial year you have been given an automatic 3.5% pay rise. If you were earning $17.60 per hour what would be your new rate?

Step 1: SIMPLE PERCENTAGE

$\frac{3.5}{100} \times 17.6 = 0.62$

Step 2: DIFFERENCE: Since it is a 3.5% pay RISE, $0.62 is added to the original: $17.60 + 0.62 = \$18.22$

C.  A new dress is now $237 reduced from $410. What is the percentage difference? As you can see, the problem is in reverse, so we approach it in reverse.

Step 1: DIFFERENCE: Since it is a discount the difference between the two is the discount. Thus we need to subtract $237.00 from $410 to see what the discount was that we received. $410 - \$237 = \$173$

Step 2: SIMPLE PERCENTAGE: now we need to calculate what percentage of $410 was $173, and so we can use this equation: $\frac{x}{100} \times 410 = 173$

We can rearrange the problem in steps: $\frac{x}{100} \times 410^{\div 410} = 173^{\div 410}$ this step involved dividing 410 from both sides to get $\frac{x}{100} = \frac{173}{410}$ Next we work to get the $x$ on its own, so we multiply both sides by 100.

Now we have $x = \frac{173}{410} \times \frac{100}{1}$ Next we solve, so 0.42 multiplied by 100, $\therefore \ 0.42 \times 100$ and we get 42.

$\therefore$ The percentage difference was 42%.

Let's check: 42% of $410 is $173, $410 - $173 = $237, the cost of the dress was $237.00 ✓.

## 12. Your Turn:

a)  GST adds 10% to the price of most things. How much does a can of soft drink cost if it is 80c before GST?

b)  A Computer screen was $299 but is on special for $249. What is the percentage discount?

c)  Which of the following is the largest? $\frac{3}{5}$ $or$ $\frac{16}{25}$ $or$ $0.065$ $or$ $63\%$? (Convert to percentages)

# 12 (continued). Activity: What do I need to get on the final exam???

|  | Grade (%) | Weight (%) |
|---|---|---|
| **Assessment 1** | 30.0% | 10% |
| **Assessment 2** | 61.0% | 15% |
| **Assessment 3** | 73.2% | 30% |
| **Assessment 4** | 51.2% | 5% |
| **Final Exam** |  | 40% |

| Final Grade | Overall Percentage Needed |
|---|---|
| High Distinction | 100 - 85% |
| Distinction | 84 - 75% |
| Credit | 74 - 65% |
| Pass | 64 - 50% |
| Fail | 49 - 0% |

1.  **How much does each of my assessments contribute to my overall percentage, which determines my final grade?**

|  | Calculation | Overall % |
|---|---|---|
| Assessment 1 | This assessment contributes a maximum of 10% to my overall percentage and I scored 30.0% of those 10%, so **30.0 ÷ 100 x 10 = 3** $\quad \frac{30}{100} \times \frac{10}{1} = 3$ | 3.0 % |
| Assessment 2 | This assessment contributes a maximum of 15% to my overall percentage and I scored 61.0% of those 15%, so |  |
| Assessment 3 |  |  |
| Assessment 4 |  |  |
|  | **Total** |  |

**Check: Your total should be 36.67%**

## 2. What do I need to score on the final exam to get a P, C, or a D? Can I still get a HD?

| | Calculation | Required score |
|---|---|---|
| **P** | For a Pass, I need to get at least 50% overall. I already have 36.67%, so the final exam needs to contribute **50 - 36.67 = 13.33**<br>The exam contributes a maximum of 40% to my overall grade, but I only need to get 13.33%, so how many percent of 40% is 13.33%?<br>**? ÷ 100 x 40 = 13.33**<br>**? = 13.33 ÷ 40% x 100**<br>**? = 33.33** | 33.33% |
| **C** | For a Credit, I need to get at least 65% overall. I already have 36.67%, so the final exam needs to contribute:<br><br><br>The exam contributes a maximum of 40% to my overall grade, but I only need to get                    , so: | |
| **D** | | |
| **HD** | | |

**Note: In most subjects, you need to score a certain percentage on the exam to pass the subject regardless of your previous results. In some cases, this may be up to 50%. Check your subject outline!**

Watch this short Khan Academy video for further explanation:

**"Percent word problem"**
https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/percent-intro-pre-alg/v/percent-word-problems

# 13. Ratio

Understanding ratio is very closely related to fractions. With ratio, comparisons are made between equal sized parts/units. The ratio symbol is ' : ' and it is used to separate the quantities being compared. For example, 1:3 means 1 unit to 3 units, and the units are the same size. Whilst a ratio can be expressed as a fraction, the ratio 1:3 is not the same as one third. For example, let's look at the rectangle below which has been divided into four equal parts.
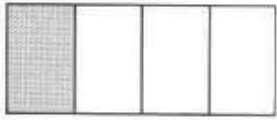
The rectangle is 1 part grey to 3 parts white. Ratio is 1:3

The rectangle is 1 part grey and a total of 4 parts. Fraction is $\frac{1}{4}$

An example to begin:  My 2 stroke mower requires petrol and oil mixed to a ratio of 1:25. This means that I add one part oil to 25 parts petrol.  No matter what unit of measure is used, the ratio must stay the same.  So if I add 200mls of oil to my tin, I add 200mls x 25 = 5000mls of petrol. Total volume = 5200mls (= to 5.2 litres). Ratio relationships are multiplicative.

Mathematically:       1 : 25  is the same as

2 : 50  is the same as

100 : 2500 is the same as

200 : 5000 and so on

**Example Problem:**

Making biscuits uses proportion. My recipe states that to make 50 biscuits, I need:
   5 cups of flour
   ½ a cup of sugar
   1 tin of condensed milk
   150g of choc chips.

How much of each ingredient will I need if I require 150 biscuits?

To calculate we need to find the ratio.  If the recipe makes 50 and we need 150 we triple the recipe
$$(150 \div 50 = 3)$$
Therefore:       5 x 3 cups of flour = 15 cups,
   ½ x 3 = 1 ½ cups of sugar
   1 x 3 =3 tins of condensed milk and
   150g x 3 = 450g of choc chips

## 13. Your Turn:
Bread is a classic ratio/proportion problem.  Standard white loaf recipe is:
   100 parts flour
   65 parts warm water
   2 parts salt
   1.5 parts yeast
How much flour is needed if:

a)   I use 50 grams of salt?

b)   I use 13 cups of water?

### 13. Your Turn (continued):

c) A tree shadow is 13m. A stick in the same vicinity is 30cm and casts a shadow of 50cm. How tall is the tree?

d) A patient is prescribed 150mg of soluble aspirin. We only have 300mg tablets on hand. How many tablets should be given?

e) A solution contains fluoxetine 20mg/5mL. How many milligrams of fluoxetine are in 40mL of solution?

f) A stock has the strength of 5000units per mL. What volume must be drawn up into an injection to give 6500units?

g) An intravenous line has been inserted in a patient. Fluid is being delivered at a rate of 42mL/h. How much fluid will the patient receive in
   a. 2 hours?
   b. 8 hours?
   c. 12 hours?

h) Penicillin syrup contains 200mg of penicillin in 5mL of water. If a patient requires 300mg of penicillin how much water will be required to make the syrup?

> Watch this short Khan Academy video for further explanation:
> **"Solve a proportion with unknown variable word problem"**
> https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/cc-7th-constructing-proportions/v/find-an-unknown-in-a-proportion-2
>
> KHANACADEMY

**Solve by applying what you have understood so far**.

i) $3 \times (9 + 5) - 4 \times 6 =$

m) $-(-3)^2 - 4 \times 1 =$

j) $100 \div 5 - 3 \times 4 + 2(9 + (-1)) =$

n) $\dfrac{(-3)+(-9)}{(-3)-(-9)} =$

k) $12 - \dfrac{3-7}{4-3\times2} =$

o) $100 - 2 \times (-7) + 3 =$

l) $(-4) - 6 \times (-3) =$

p) $\dfrac{(-30)-2\times(-5)}{1+(-13)\times(-3)} =$

# 14. Averages

When examining a collection of data, it is most common to find a measure of the "middle" value. The three common **measures of central tendency** are:
- Mean
- Median
- Mode

**Mean:** The most commonly used measure; often referred to as the 'average'. It is calculated by adding up all of the data and dividing by how many pieces of data you had. For example, the average temperature over one month might be calculated using mean. We would add up the temperature for each day and then divide the result by the number of days for the month. This would then give us a 'mean' average for temperature for that month. Other factors may be considered too when collecting the type of data required, such as the time of day, the highest temperature or the lowest temperature.

The formula for mean is: $\bar{x} = \frac{\Sigma x}{n}$
- $\bar{x}$ is called "$x$ bar" and represents mean.
- $\Sigma$ is called "sigma" and means sum or add up
- $x$ is the individual pieces of data or observations
- $n$ is how many pieces of data or the total number of observations
- Hence, $(\bar{x} = \frac{\Sigma x}{n})$ $The\ highest\ temp\ for\ July\ mean = \frac{the\ sum\ of\ the\ highest\ temp\ for\ each\ day}{the\ number\ of\ days\ in\ July}$

**Median:** The middle number when the data is arranged in order from lowest to highest. One way to remember this is to think about the 'median strip' in the road. The median is a measure of central tendency often used to discuss the average value of real-estate in certain districts. For example, seven houses sold in a suburb in July, for the following prices: $450,000; $500,000; $420,000; $485,000; $445,000; $950,000; $425,000. We arrange these figures in order and then take the middle figure. This is often a more accurate representation than a mean average because the figures at the high or low end may be significantly different, thereby affecting the end result. In this example the median average would be $450,000; whereas the mean average would be approximately $530,000 $\left(\bar{x} = \frac{\Sigma x}{n}\right)$; quite a difference.

**Mode:** The most common number. We can have more than one mode in a data set. For example, a bimodal data set would have two modes. Mode is useful in situations where identifying the most common number is important in decision-making. For example, to purchase chairs for a classroom, the most common student height would be used.

## 14. Your Turn

The following is a data set that records one player's point score in seven consecutive games of basketball.

30, 80, 56, 22, 22, 36, 39

Calculate the three different measures of central tendency:
a) Mean:
b) Median:
c) Mode:
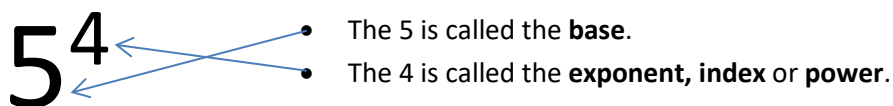d) Which measure of central tendency is the most meaningful?

# 15. Powers

Powers are a method of simplifying expressions.

- An equation such as : $7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 63$ could be simplified as: $7 \times 9 = 63$
- Whereas an expression such as: $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ could be simplified as: $7^9$

A simple way to describe powers is to think of them as how many times the base number is multiplied by itself.

- $5 \times 5 \times 5 \times 5$ is the same as $5^4$ is the same as 625

$$5^4$$

- The 5 is called the **base**.
- The 4 is called the **exponent, index** or **power**.

- The most common way to describe this expression is, 'five to the power of four.'
- The two most common powers (2 & 3) are given names: $3^2$ is referred to as '3 squared' and $3^3$ as '3 cubed.'

$3^2$   $3^3$
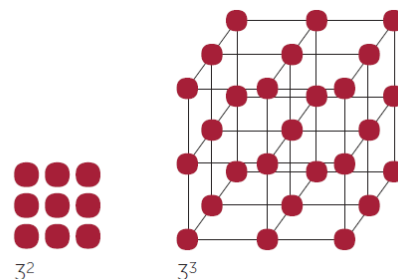
A **negative power**, for example $6^{-3}$, is the same as $\frac{1}{6^3}$ : it is the **reciprocal**.

The most common base is 10. It allows very large or small numbers to be abbreviated. This is commonly referred to as scientific notation.

- $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$. (Note: 100 000 has 5 places (as in place value) behind '1', a way to recognise 10 to the power of five
- $10^{-4} = \frac{1}{10^4} = 10 \div 10 \div 10 \div 10 = 0.0001$. (Note: 0.0001 has 4 places (as in place value) in front; one divided by 10 to the power of four)
- $3000 = 3 \times 1000 = 3 \times 10^3$
- $704,500,000 = 7.045 \times 100,000,000 = 7.045 \times 10^8$
- A classic example of the use of scientific notation is evident in the field of chemistry. The number of molecules in 18 grams of water is 602,000,000,000,000,000,000,000. This is simplified as $6.02 \times 10^{23}$ which is much easier to read.
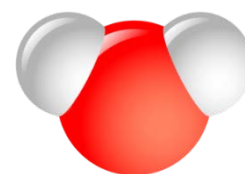
## 15. Your Turn:

Write the following in scientific notation:

a) 450
b) 90000000
c) 3.5
d) 0.0975

Write the following numbers out in full:

e) $3.75 \times 10^2$
f) $3.97 \times 10^1$
g) $1.875 \times 10^{-1}$
h) $(-8.75) \times 10^{-3}$

# 16. Power Operations

- Some basic rules :
  - Any base number to the power of 1 is equal to the base number, for example: $5^1 = 5$
  - Any base number to the power of 0 equals 1, so: $4^0 = 1$
  - Powers can be simplified if they are <u>multiplied</u> or <u>divided</u> and have the <u>same</u> base.
  - Powers of powers are multiplied. Hence, $(2^3)^2 = 2^3 \times 2^3 = 2^6$
    - $(2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6$
  - A negative power indicates a reciprocal: $3^{-2} = \frac{1}{3^2} \therefore \frac{1}{9}$
- $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 = 2^5 \therefore 32$
- Therefore, when <u>multiplying</u> powers with the same base, we <u>add</u> the indices. $2^3 \times 2^2 = 2^{3+2} = 2^5$
- This also applies to division. $4^4 \div 4^2 = 4^{(4-2)} = 4^2 \therefore 16 \quad (256 \div 16 = 16)$

**Example Problems:**

1. Simplify $6^5 \times 6^3 \div 6^2 \times 7^2 + 6^4 =$

$$6^{5+3-2} \times 7^2 + 6^4 =$$

$$6^6 \times 7^2 + 6^4$$

2. Simplify $g^5 \times h^4 \times g^{-1} =$

$$g^{5+(-1)} \times h^4 =$$

$$g^4 \times h^4$$

**16. Your Turn:**

Simplify the following:

a) $5^2 \times 5^4 + 5^2 =$

b) $x^2 \times x^5 =$

c) $4^2 \times t^3 \div 4^2 =$

d) $(5^4)^3 =$

> This is a summary of some index laws for your reference:
>
> $a^n \times a^m = a^{n+m}$
>
> $\frac{a^n}{a^m} = a^{n-m}$
>
> $(a^n)^m = a^{nm}$
>
> $a^{-n} = \frac{1}{a^n}$
>
> $a^{\frac{1}{n}} = \sqrt[n]{a}$
>
> $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$
>
> $a^0 = 1$

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Watch this short Khan Academy video for further explanation:
**"Simplifying expressions with exponents"**
https://www.youtube.com/user/khanacademy

# 17. Roots

Previously we have looked at powers: For example, $4^2 = 16$

- A root is used to find an unknown base. For example, $\sqrt{16} = 4$
- In words, $\sqrt{16}$ is expressed as, 'what number multiplied by itself equals 16?'

Like exponents, the two most common roots (2 & 3) are expressed as the 'square root' and the 'cube root' respectively.

- $\sqrt{64}$ is expressed as the square root of 64. $\sqrt[3]{27}$ is expressed as the cube root of 27 (note square root does not have a 2 at the front like the cube root does; the 2 is assumed).

Let's look at the relationship between 3 and 27. We know that 3 cubed is 27 ($3 \times 3 \times 3$).

- So, 3 is the cube root of 27; $\sqrt[3]{27} = 3 \ and \ 3^3 = 27$

Now let's look at negative numbers: $3^3 = 27 \ and \ (-3)^3 = -27$

- Yet, $5^2 = 25$ and $(-5)^2 = 25, \therefore \ -5$ is the square root of 25.
- Hence, $\sqrt{25} = \pm 5$

Simplifying roots is difficult without a calculator. This process requires an estimation of the root of a number. To estimate a root, it is helpful to know the common powers. Some common powers are given in the table below:

| $n^2$ | | $\sqrt{}$ | |
|---|---|---|---|
| $1^2 =$ | 1 | $\sqrt{1} =$ | $\pm 1$ |
| $2^2 =$ | | | |
| $3^2 =$ | | | |
| $4^2 =$ | | | |
| $5^2 =$ | | | |
| $6^2 =$ | | | |
| $7^2 =$ | | | |
| $8^2 =$ | | | |
| $9^2 =$ | 81 | $\sqrt{81} =$ | $\pm 9$ |
| $10^2 =$ | | | |
| $11^2 =$ | | | |
| $12^2 =$ | | | |
| $13^2 =$ | | | |

**17. Your Turn:**

Complete the table to the left.

Using the table:

$\sqrt{81} = \pm 9$ (9 squared is 81)

$\sqrt{56} = ?$ An example as such requires estimation. Look to the table, if the square root of 49 is 7, and the square root of 64 is 8, then the square root of 56 is between 7 and 8 (7.48).
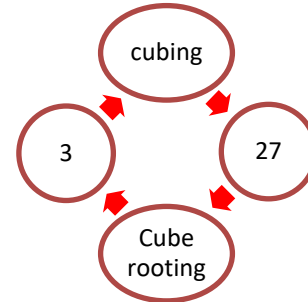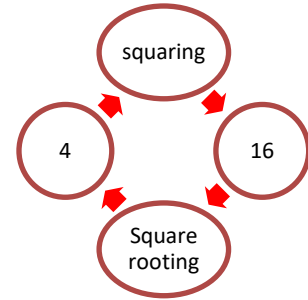
A **surd** is a special root which cannot be simplified into a whole number. For instance, $\sqrt{4} = \pm 2$, 2 is a whole number; therefore, $\sqrt{4}$ is not a surd. However, $\sqrt{3} = 1.732$, 1.732 is not a whole number; therefore, $\sqrt{3}$ is a surd.

Large roots, such as $\sqrt{56}$ must be simplified to determine if they are surds. This process is explained on the next page.

# 18. Root Operations

Some basic rules:

- $\sqrt{x} \times \sqrt{y} = \sqrt{x \times y}$      With numbers: $\sqrt{6} \times \sqrt{4} = \sqrt{6 \times 4} = \sqrt{24}$
- $\sqrt{x^2} = x$      With numbers: $\sqrt{5^2} = 5$

Simplifying roots requires finding factors which are square numbers (you have done this by finding some factors in the table on the previous page). Factors are all of the whole numbers that can be divided exactly into a number. For example, the factors of 12 are: 1, 2, 3, 4, 6, 12.

- Let's simplify $\sqrt{56}$ ; 56 has multiple factors and it is good practice to list them all:
  * $1 \times 56$
  * $2 \times 28$
  * $4 \times 14$    ($4 \times 14$ *is* the key since it has a square number which is 4.)
  * $7 \times 8$
    So, we can simplify $\sqrt{56} = \sqrt{4} \times \sqrt{14} = 2\sqrt{14}$ (because $\sqrt{4} = 2$)

## Example Problems:

1. Simplify $\sqrt{32}$

   $\sqrt{32}$    32 has the factors: $1 \times 32, 2 \times 16, 4 \times 8$. The biggest square number in this list is 16.

   $\therefore \sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

2. Simplify $5\sqrt{18}$

   $5\sqrt{18}$    18 has the factors: $1 \times 18, \; 2 \times 9, \; 3 \times 6$. The biggest square number is 9.

   $\therefore 5\sqrt{18} = 5 \times \sqrt{9} \times \sqrt{2}$

   $\qquad = 5 \times 3 \times \sqrt{2}$

   $\qquad = 15\sqrt{2}$

## 18. Your Turn:

a) Simplify $\sqrt{81}$

b) Simplify $\sqrt{72}$

c) Simplify $2\sqrt{48}$

d) Simplify $4\sqrt{338}$

Watch this short Khan Academy video for further explanation:
**"Simplifying square roots"**
https://www.khanacademy.org/math/algebra-basics/core-algebra-foundations/square-roots-for-college/v/simplifying-square-roots-1

# 19. Fraction Powers/Exponents

Another way to express the cube root or square root is to use fraction powers. For example, an expression $9^{\frac{1}{2}}$ is also the same as the square root of nine: $\sqrt{9}$. Similarly, $\sqrt{16}$ could be expressed as $16^{\frac{1}{2}}$.

Then the same applies for the fractional power/exponent $\frac{1}{3}$, for example, $\sqrt[3]{8}$ can also be expressed as $8^{\frac{1}{3}}$

Thus we could say that an exponent such as $\frac{1}{x}$ implies to take the $x$ th root.

Therefore, to generalise we could say that if $x^{\frac{1}{2}} = \sqrt{x}$ and $x^{\frac{1}{3}} = \sqrt[3]{x}$ then $x^{\frac{1}{n}} = \sqrt[n]{x}$

How does this work:

Well let's look at $27^{\frac{1}{3}}$. What can we say about this expression?

- ° It is twenty seven to the fractional exponent of a third
- ° It implies the cube root of 27
- ° The cube root of 27 is 3. $(3 \times 3 \times 3) = 27$
- ° Thus we also know that $3^3 = 27$
- ° Now we could simplify $27^{\frac{1}{3}}$ to $(3^3)^{\frac{1}{3}}$
- ° So using our knowledge of index laws (page 21), we could also say that $(3^3)^{\frac{1}{3}}$ is the same as $3^{(3 \times \frac{1}{3})}$
- ° This then cancels out to $3^1 = 3$

This is a long way to explain how fractional powers/exponents work, but as with all mathematics, with practise a pattern forms making the 'how and why' of mathematical ideas more evident.

**What about negative fractional powers/exponents?**

Remember we worked with these briefly in section 15, where we looked at $6^{-3}$, is the same as $\frac{1}{6^3}$: it is the reciprocal. Hence we can generalise that $a^{-n} = \frac{1}{a^n}$

Let's explore:

- ° $(27)^{-\frac{1}{3}}$
- ° If $a^{-n} = \frac{1}{a^n}$ then $(27)^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}}$
- ° So we know that $27^{\frac{1}{3}} = \sqrt[3]{27}$
- ° $\therefore (27)^{-\frac{1}{3}} = \frac{1}{3}$

## 19. Your Turn:

Evaluate:

- a) $(81)^{\frac{1}{2}}$
- b) $(81)^{-\frac{1}{2}}$

# 20. Logarithms

With roots we tried to find the unknown base. For instance, $x^3 = 64$ is the same as $\sqrt[3]{64} = x$; ($x$ is the base).

A **logarithm** is used to find an unknown **power/exponent**. For example, $4^x = 64$ is the same as $\log_4 64 = x$

This example above is spoken as: 'The logarithm of 64 with base 4 is $x$.' The base is written in subscript.

The general rule is: $N = b^x \Leftrightarrow \log_b N = x$

- In mathematics the base can be any number, but only two types are commonly used:
  - $\log_{10} N$ $(base\ 10)$ is often abbreviated as simply Log, and
  - $\log_e N$ $(base\ e)$ is often abbreviated as Ln or natural log
- $\log_{10} N$ is easy to understand for: $\log_{10} 1000 = \log 1000 = 3$ $(10^3 = 1000)$
$$\log 100 = 2$$
- Numbers which are not 10, 100, 1000 and so on are not so easy. For instance, $\log 500 = 2.7$ It is more efficient to use the calculator for these types of expressions.

## 20. Your Turn:

Write an exponential equation for the following:

a) $3 = \log_2 8$

b) $\frac{1}{2} = \log_{25} 5$

c) $\log_7 49 = 2$

d) $\frac{3}{2} = \log_{16} 64$

e) $\log_8 \frac{1}{4} = -\frac{2}{3}$

Use a calculator to solve the following:

f) $\log 10000 =$

g) $\log 350 =$

h) $\ln 56 =$

i) $\ln 100 =$

---

Watch this short Khan Academy video for further explanation:
**"Logarithms"**
https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/logarithms

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Some resources to help clarify using logs:
- **Using logs in the real world**

http://betterexplained.com/articles/using-logs-in-the-real-world/

- **TED talk: Logarithms explained**

http://ed.ted.com/lessons/steve-kelly-logarithms-explained

# 21. Unit Conversions

Measurement is used every day to describe quantity. There are various types of measurements with various units of measure. For instance, we measure time, distance, speed, weight and so on. Then time is measured in seconds, minutes, hours, weeks, days, months, years...

Often we are required to convert a unit of measure; we may be travelling and need to convert measurements from imperial to metric, or we may need to convert millimetres to metres for ease of comparison. In the fields of science and medicine, converting measurement can be a daily activity.

It helps to apply a formula to convert measurement. However, it is essential to understand the how and why of the formula otherwise the activity becomes one we commit to memory without understanding what is really happening. More mistakes are made when procedures are carried out without understanding, which could be particularly troublesome in the field of medicine.

The metric system is base 10 which makes unit conversion relatively easy. For example, we know that there are 10mm in a cm, and there are 100cm in a metre. The metre is a standard unit of **length** and it helps to understand what a metre looks like so as you get a 'feel' for measurement. The height of your kitchen bench might be approximately a metre, and the width of your little finger might be equivalent to a centimetre.

Let's look at conversions symbolically.

$1m = 100cm = 1000mm$ or we could say that $1mm = \frac{1}{10}cm = \frac{1}{1000}m$

**Unit Conversion rules:**

° Always write the unit of measure associated with every number.
° Always include the units of measure in the calculations.

Unit conversion requires algebraic thinking which will be covered in the next booklet; however, here you will be introduced to the concept. Let's convert 58mm into metres. $58mm \times \frac{1m}{1000mm} = 0.058m$

The quantity $\frac{1m}{1000mm}$ is called a **conversion factor;** it is a division/quotient; in this case it has metres on top and $mm$ on the bottom. Another name for the conversion factor is a solution map. So we can see that we work with information given, a conversion factor, and then the desired unit.

info given ➡ conversion factor ➡ desired unit
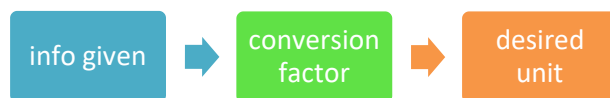
**Example problem:**

Convert 32 centimetres to metres. There are 100cm in a metre so our solution map is $\frac{1m}{100cm}$

The working is as follows: $32\cancel{cm} \times \frac{1m}{100\cancel{cm}} = 0.32m$

Here we can cancel out the cm and are left with metres and the new just need to divide 32 by 100 to get 0.32m.

It is helpful to have a thinking process to follow. This one comes from the book, *Introductory Chemistry* (Tro, 2011, pp. 25-35). There are four steps to the process: sort, strategise, solve and check.

- ° **Sort:** First we sort out what given information is available.
- ° **Strategise**: The second step is where a solution map is created. You may need to look at a conversion table and possibly use a combination of solution maps.
- ° **Solve**: This is the third step which simply means to solve the problem.
- ° **Check**: The last step requires some logical thought; does the answer make sense?

**Example problem:** Convert 2 kilometres (km) into centimetres (cm).

- ° **Sort**: we know there are 1000 metres in one km, and 100cm in one metre.
- ° **Strategise**: So our maps could be $\frac{1000m}{1km}$ $and$ $\frac{100cm}{1m}$
- ° **Solve**: $2km \times \frac{1000m}{1km} \times \frac{100cm}{1m} = x\ cm$

$$2\cancel{km} \times \frac{1000\cancel{m}}{1\cancel{km}} \times \frac{100cm}{1\cancel{m}} = 2 \times 1000 \times 100\ cm \ \therefore 2km = 200,000cm$$

- ° **Check**: is there 200,000cm in a kilometre? Yes that seems sensible.

**21. Your Turn:**

Convert the following:

a) 285m into kilometres

- ° Sort

- ° Strategise

- ° Solve

- ° Check

b) 96cm into kilometres

- ° Sort

- ° Strategise

- ° Solve

- ° Check

c) Using this information: $1\ m^2 = 10000cm^2 = 1000000mm^2$
   Convert $1.5m^2\ into\ mm^2$
- ° Sort

$1m = 100cm = 1000m$

- ° Strategise

- ° Solve

- ° Check

The symbol SI comes from the initials of the French term: *Systeme International d'Unites* which means international unit system. This system consists of seven base units:  metre, kilogram, second, ampere, kelvin, candela and mole.

| Quantity | Name | Symbol |
|---|---|---|
| length | meter (or metre) | m |
| mass | kilogram | kg |
| time | second | s |
| electric current | ampere | A |
| thermodynamic temperature | kelvin | K |
| luminous intensity | candela | cd |
| amount of substance | mole | mol |

### HERE IS A METRIC CONVERSION TABLE:

| Prefix | Symbol | | Multiples & Submultiples | | Measure | Unit |
|---|---|---|---|---|---|---|
| mega | M | million | 1, 000,000 | $10^6$ | information | Mb = megabyte |
| kilo- | k | thousand | 1,000 | $10^3$ | mass | kg = kilogram |
| | | | | | length | km = kilometre |
| deci- | d | tenth | 1/10 = 0.1 | $10^{-1}$ | length | dm = decimetre |
| centi- | c | hundredth | 1/100 = 0.01 | $10^{-2}$ | length | cm = centimetre |
| milli- | m | thousandth | 1/1000 = 0.001 | $10^{-3}$ | mass | mg = milligram |
| micro- | µ | millionth | 1/1, 000,000 = 0.000001 | $10^{-6}$ | mass | µg = microgram |
| nano- | n | billionth | 1/1,000,000,000 = 0.000000001 | $10^{-9}$ | length | nm = nanometres |

Thus, 1000 milligrams make a gram, and 1000 grams make a kilogram. So how many milligrams are in a kilogram?

$$\frac{1000\ g}{1} \times \frac{1000 mg}{1g} = 1\ 000\ 000\ mg$$

$$\frac{1000\ \cancel{g}}{1} \times \frac{1000 mg}{1\cancel{g}} = 1\ 000\ 000\ mg$$
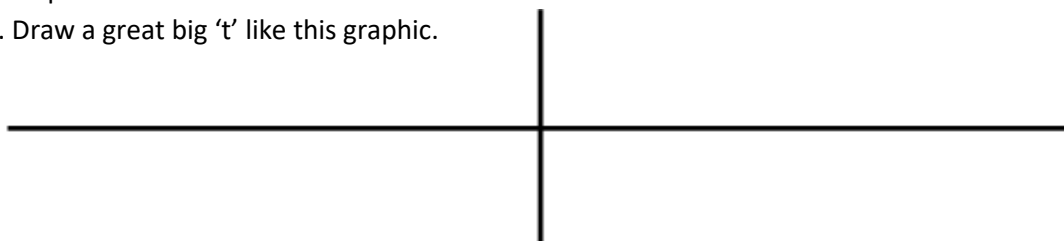
### HERE IS A VOLUME CONVERSION TABLE:

| SI unit | Liquid volume measure |
|---|---|
| 1 $m^3$ = cubic metre | 1000 Litres (L) |
| 1 $dm^3$ = cubic decimetre | 1 Litre (L) |
| 1 $cm^3$ = cubic centimetre | 1 millilitre (mL) |

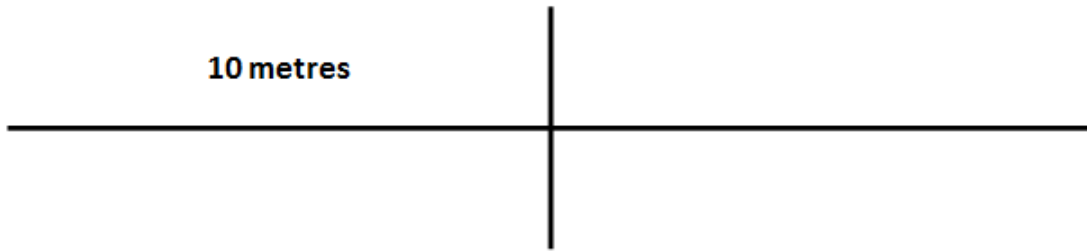### MORE STEP-BY-STEP GUIDES TO CONVERTING UNITS:

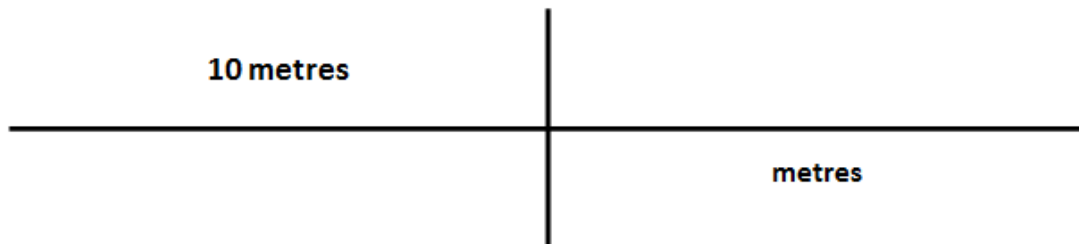**Method 1:** using a graphic to help.

Example: convert 10m into centimetres.

1. Draw a great big 't' like this graphic.

2. Put the number that you have to convert into the top left corner of this graphic.

**10 metres**

3. Put the unit of that number in the bottom right part of the graphic.

**10 metres**

metres

4. Write the unit you want in the top right part of the graphic.

**10 metres**                                          centimetres

metres

5. Write the conversion factor (solution map) in front of the units from step 3 and 4. There are 100 centimetres in a metre 100cm/1m.

**10 metres**                                          100centimetres

1metres

6. Once you have the graphic filled out, all you have left to do is multiply all the numbers on the top together, and divide this by the product of the numbers at the bottom. The unit of the answer will be 'centimetres' as the 'metres' cancel out:

$$\frac{10m \times 100cm}{1m} = 1000\ cm$$

**Method 2**

1. Write the conversion as a fraction.

2. Multiply out (include all units in the working).

3. Cancel any units that are both top and bottom.

Examples:

a) Convert 0.15g to milligrams

$$\frac{0.15\ g}{1} \times \frac{1000mg}{1g} = 150\ mg$$

b) Convert 5234 ml to litres

$$\frac{5234mL}{1} \times \frac{1L}{1000mL} = 5.234\ L$$

c) Convert 21g/L to mg/ml

$$\frac{21g}{1L} \times \frac{1000mg}{1g} \times \frac{1L}{1000mL} = 21\ mg/mL$$

## 21. Your turn (continued)

Define the following SI unit prefixes, in words, as a number, and in exponential notation:

d) kilo

e) centi

f) mega

g) deci

Convert the following:

h) How many millilitres are in a cubic metre?

i) How many inches in 38.10cm (2.54cm = 1 inch)

j) How many centimetres in 1.14 kilometres?

k) How many litres are in $3.5 \times 10^5$ millilitres?

# 22. ANSWERS

1. Introduction (pages 3 and 4)

a. F    b. T    c. T    d. T    e. F    f. six thousand    g. one hundredth    h. 15    i. 70    j. i) 4    ii) -21    iii) -9


2. Rounding and Estimating (page 5)

A. a. 34.599  b. 56.673        B .a. $\approx$ 35x60=2100    b. $\approx$ 36-13=23   c.$\approx$ 30x4=120


3. Order of Operations (page 6)

a. 31        b. 34    c. 22.4    d. 32


4. Naming Fractions (page 7)

a. $\frac{3}{16}$    b. $\frac{1}{16}$    c. $\frac{3}{16}$    d. $\frac{1}{4}$


5. Equivalent Fractions (page 8)

a. 6    b. 63    c. 27    d. 16    e. $\frac{3}{32}$    f. $\frac{1}{32}$


6. Converting Mixed Numbers to Improper Fractions (page 9)

a. $\frac{9}{2}$    b. $\frac{16}{3}$    c. $\frac{38}{5}$    d. $\frac{17}{8}$


7. Converting Improper Fractions to Mixed Numbers (page 10)

a. $1\frac{2}{5}$    b. $1\frac{3}{9} = 1\frac{1}{3}$    c. $5\frac{8}{9}$    d. $3\frac{6}{7}$


8. Converting Decimals into Fractions (page 11)

a. $\frac{65}{100} = \frac{13}{20}$        b. $2\frac{666}{1000} \approx= 2\frac{2}{3}$        c. $\frac{54}{100} = \frac{27}{50}$        d. $3\frac{14}{100} = 3\frac{7}{50}$


9. Converting Fractions into Decimals (page 12)

a. 0.739        b. 0.069        c. 56.667        d. 5.8

10. Fraction Addition and Subtraction (page 13)

a. $\frac{11}{15}$

b. $\frac{3}{4} + \frac{2}{7} = \frac{(3\times7)+(2\times4)}{(4\times7)} = \frac{21+8}{28} = \frac{29}{28} = 1\frac{1}{28}$

c. $2\frac{2}{3} + 1\frac{3}{4} = \frac{8}{3} + \frac{7}{4} = \frac{(8\times4)+(7\times3)}{(3\times4)} = \frac{32+21}{12} = \frac{53}{12} = 4\frac{5}{12}$    Note: Convert mixed to improper.

d. $6\frac{1}{24}$

e. $\frac{9}{12} - \frac{1}{3} = \frac{(9\times3)-(1\times12)}{(12\times3)} = \frac{27-12}{36} = \frac{15}{36} = \frac{5}{12}$   Note: equivalent fraction

f. $\frac{1}{3} - \frac{1}{2} = \frac{(1\times2)-(1\times3)}{(3\times2)} = \frac{2-3}{6} = \frac{-1}{6} = -\frac{1}{6}$   Note: you have taken more then you originally had.

11. Fraction Multiplication and Division (page 15)

a. $\frac{5}{12}$    b. $\frac{14}{39}$    c. $\frac{7}{27}$    d. $7\frac{1}{2}$    e. $1\frac{1}{14}$    f. $\frac{108}{175}$    g. $-\frac{1}{2}$    h. $\frac{63}{8}$ or $7\frac{7}{8}$    i. 4    j. 8

12. Percentage (page 16, 17 & 18)

a. 88c    b.. 16.7%    c. $\frac{16}{25}$
What do I need to get on my final exam? Credit (70.83%); Distinction (95.83%); HD (120.83 ∴not possible)

13. Ratios (pages 19 and 20)

a. 2500 grams    b. 20 cups    c. 7.8m    d. ½ tablet    e. 160mg    f. 1.3    g. 84mL, 336mL, 504mL    h. 7.5mL

Extra Equations

i. 18    j. 24    k. 10    l. 14    m. -13    n. -2    o. 117    p. $-\frac{1}{2}$

14. Averages (page 21)

a. 41    b. 36    c. 22    d. mean (depending on the purpose, who, what, why)

15. Powers (page 22)

a. $4.50 \times 10^2$    b. $9.0 \times 10^7$    c. 3.5    d. $9.75 \times 10^{-2}$    e. 375    f. 39.7    g. 0.1875    h. $-0.00875$

16. Power Operations (page 23)

a. $5^6 + 5^2$    b. $x^7$    c. $t^3$    d. $5^{12}$

## 17. Roots (page 24)

| $n^2$ | | $\sqrt{}$ | $\pm$ |
|---|---|---|---|
| $1^2$ | 1 | $\sqrt{1}$ | 1 |
| $2^2$ | 4 | $\sqrt{4}$ | 2 |
| $3^2$ | 9 | $\sqrt{9}$ | 3 |
| $4^2$ | 16 | $\sqrt{16}$ | 4 |
| $5^2$ | 25 | $\sqrt{25}$ | 5 |
| $6^2$ | 36 | $\sqrt{36}$ | 6 |
| $7^2$ | 49 | $\sqrt{49}$ | 7 |
| $8^2$ | 64 | $\sqrt{64}$ | 8 |
| $9^2$ | 81 | $\sqrt{81}$ | 9 |
| $10^2$ | 100 | $\sqrt{100}$ | 10 |
| $11^2$ | 121 | $\sqrt{121}$ | 11 |
| $12^2$ | 144 | $\sqrt{144}$ | 12 |
| $13^2$ | 169 | $\sqrt{169}$ | 13 |

## 18. Root Operations (page 25)

a. 9     b. $6\sqrt{2}$     c. $8\sqrt{3}$     d. $52\sqrt{2}$

## 19. Fraction Powers (page 26)

a. $\sqrt{81} = \pm9$     b. $\frac{1}{\sqrt{81}} = \frac{1}{\pm9} = \pm\frac{1}{9}$

## 20. Logarithms (page 27)

a. $3 = \log_2 8 \leftrightarrow 2^3 = 8$

b. $\frac{1}{2} = \log_{25} 5 \leftrightarrow 25^{\frac{1}{2}} = 5$

   $25^{\frac{1}{2}} \leftrightarrow \sqrt{25}$

c. $\log_7 49 = 2 \leftrightarrow 7^2 = 49$

d. $\frac{3}{2} = \log_{16} 64 \leftrightarrow 16^{\frac{3}{2}} = 64$

   $16^{\frac{3}{2}} = 16^{3\times\frac{1}{2}}$

   $Thus \sqrt{16^3} = \sqrt{4096} = 64$

e. $\log_8 \frac{1}{4} = -\frac{2}{3} \leftrightarrow 8^{(-\frac{2}{3})} = \frac{1}{4}$

   $\frac{1}{8^{\frac{2}{3}}} = \frac{1}{8^{2\times\frac{1}{3}}}$

   $= \frac{1}{\sqrt[3]{8^2}} = \frac{1}{\sqrt[3]{64}}$

   $= \frac{1}{4}$

f. 4

g. 2.54

h. 4.025

i. 4.605

## 21. Unit Conversions (page 29)

a. $0.285km$     b. $0.00096km$ or $9.6km \times 10^{-4}$     c. $1500000mm^2$ or $1.5mm^2 \times 10^6$

21. Unit Conversions (continued)  (page 32)

d. kilo    one thousand, 1 000, $10^3$

e. centi   one hundredth, 0.01, $10^{-2}$

f. mega    one million, 1 000 000, $10^6$

g. deci    one tenth, 0.1, $10^{-1}$

h.  $1.00 \times 10^6 mL$    $\frac{1m^3}{1} \times \frac{1000L}{1m^3} \times \frac{1000mL}{1L} = 1000\,000mL$ or $1.00 \times 10^6 mL$

i.   15 inches:   if 2.54cm = 1 inch, then    $\frac{38.1cm}{1} \times \frac{1inch}{2.54cm} = 15 inches$

| 38.1cm | 1 inch |
|---|---|
|  | 2.54cm |

j.   $1.14 \times 10^5 cm$:    $\frac{1.14km}{1} \times \frac{100cm}{1m} \times \frac{1000m}{1km} = 114000cm$

| 1.14km | 100 cm | 1000m |
|---|---|---|
|  | 1 m | 1km |

k.   350litres :    $\frac{350000mL}{1} \times \frac{1L}{1000mL} = 350L$

| 350000mL | 1litre |
|---|---|
|  | 1000mL |

# 23. Glossary

**Addition** Bringing two or more numbers (or things) together to make a new total, e.g. 3+2

**Arithmetic** The study of numbers and their manipulation

**Average** A calculated "central" value of a set of numbers (see mean)

**Denominator** The bottom number in a fraction; it shows how many equal parts the item is divided into

**Difference** The result of subtracting one number from another (how much one number differs from another); e.g. the difference between 8 and 3 is 5

**Division** Splitting into equal parts or groups, e.g. 12 chocolates shared by 3 friends are divided into 4 chocolates per person

**Equation** An equation says that two things are the same, using mathematical symbols, e.g. 7 + 2 = 10 - 1

**Equivalent fraction** Alternative ways of writing the same fraction, e.g. 1/2 = 2/4 = 3/6; equivalent fractions have the same value, even though they look different

**Exponent** The power to which a number is raised. The exponent of a number says how many times to use that number in a multiplication and is written as a small number to the right and above the base number, e.g. $8^3$= 8 x 8 x 8 = 512

**Fraction** A quantity that is not a whole number, e.g. 1/2, 3/7; (a part of a whole)

**Improper fraction** A fraction with the numerator (top number) larger than (or equal to) the denominator (bottom number), e.g. 3/2, 7/3, 16/15

**Integer** Positive and negative whole numbers including zero, e.g. -6, -2, 0, 1, 12

**Irrational number** A real number that cannot be written as a simple fraction, e.g. π = 3.14158…(Pi cannot be turned into a fraction)

**Logarithm** How many of one number need to be multiplied to get another number, e.g. how many 2s need to be multiplies to get 8? 2 x 2 x 2 = 8, so we need to multiply 3 of the 2s to get 8, so the logarithm is 3

**Mean** A calculated "central" value of a set of numbers (often called average); to calculate, add up all the numbers, then divide by how many numbers there are

**Median** The middle number in a sorted list of numbers

**Mixed fraction (also called mixed numbers)** A whole number and a proper fraction combined, e.g. 1 1/3 (one and three quarters)

**Mode** The number that appears most often in a set of numbers

**Multiplication** A quicker way of repeating addition, e.g. 5 x 3 = 5 + 5 + 5 = 15

**Numerator** The top number in a fraction; it shows how many parts we have

**Operator** Symbols used between numbers to indicate a task or relationship, e.g. +, -, x, =

**Percent** Part per 100; it tells you a ratio out of 100, e.g. 25% means 25 per 100, so when 25% of people have ice cream that means that 25 out of every 100 people have ice cream. If there were 500 people, then 125 would have ice cream

**Power** see Exponent

**Product** The answer when two or more numbers are multiplied together, e.g. 18 is the product of 6 and 3

**Proper fraction** A fraction where the numerator (the top number) is less than the denominator (the bottom number), e.g. 1/4, 5/6

**Quotient** The answer after you divide one number by another (dividend ÷ divisor = quotient); e.g. in 12 ÷ 3 = 4, 4 is the quotient

**Ratio** A ratio shows the relative sizes of two or more values, a ratio can be shown in different ways, e.g. if there is 1 boy and 3 girls you could write the ratio as: 1:3, 1/4 are boys and 3/4 are girls, 0.25 are boys (by dividing 1 by 4), or 25% are boys (0.25 as a percentage)

**Rational number** A real number that can be written as a simple fraction, e.g. 1.5 (because 1.5 = 3/2)

**Rounding** Reducing the digits in a number while trying to keep its value similar; the result is less accurate, but easier to use, e.g. 73 rounded to the nearest ten is 70 because 73 is closer to 70 than to 80.

**Subtraction** Taking one number away from another, e.g. 5 - 3

**Sum** The result of adding two or more numbers, e.g. 9 is the sum of 2, 4 and 3

**Square root** A value that, when multiplied by itself, gives the number, e.g. 4 x 4 = 16, so the square root of 16 is 4

**Unit** A quantity used as a standard of measurement, e.g. units of time are second, minute, hour, day, week, moth, year and decade

# 24. Helpful websites

Integers:	http://www.factmonster.com/ipka/A0876848.html

Rounding:	http://www.mathsisfun.com/rounding-numbers.html

Estimating:	http://mathandreadinghelp.org/how_to_estimate_a_math_problem.html

Order of Operations: http://www.mathgoodies.com/lessons/vol7/order_operations

Equivalent Fractions: http://www.mathsisfun.com/equivalent_fractions.html

Mixed Fraction: Jamit Way Back Machine

Decimals:	http://www.mathsisfun.com/converting-fractions-decimals.html

Addition Subtraction: http://www.mathsisfun.com/fractions_addition.html

Multiplication Division: http://www.mathsisfun.com/fractions_multiplication.html

Ratio:	http://www.mathsisfun.com/numbers/ratio.html


Nursing Calculations: http://nursing.flinders.edu.au/students/studyaids/drugcalculations/

Percentage:	https://percentagecalculator.net/
	http://www.mathsisfun.com/percentage.html

Powers:	https://www.mathsisfun.com/definitions/power.htmlind.htm

Roots:	http://www.math.utah.edu/online/1010/radicals/

Logarithms:	http://www.mathsisfun.com/algebra/logarithms.html